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MINIMUM-POWER ARRAY PROCESSING OF THE TFO LONG-NOISE SAMPLE

ADVANCED ARRAY RESEARCH
Special Report No. 12

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ABSTRACT

This report investigates the effectiveness of the minimum-power array processing technique in determining seismometer inequalizations. The technique involves partitioning the seismometer array into two groups and designing MCF's for each group so that the mean-square-error between the two MCF outputs is a minimum under the constraint that the output power of one of the MCF's is unity. The two MCF sets are known as the group-coherence filters; the difference between these sets is known as the minimum-power array processor.

Estimates of the noise wavenumber spectrum from the wavenumber responses of the group-coherence filters are distorted due to seismometer inequalization; however, a more reasonable estimate of the noise wavenumber spectrum from the wavenumber response of the minimum-power array processor should be possible because the minimas in the processor's wavenumber response correspond to the wavenumber regions where the wavenumber responses of the two group-coherence MCF's are very similar (e.g., at the peaks of the noise wavenumber power spectrum).

Seismometer inequalization was to be determined from the adjustment in weight and phase required for each filter so that the wavenumber responses of the group-coherence MCF's would agree with a reasonable noise wavenumber spectrum. However, results from the TFO long-noise sample and two synthetic models show that the technique, although excellent for generating maximum coherent channels, lacks the wavenumber resolution desired for studying seismometer inequalization. This latter conclusion is at least true for small arrays such as TFO.

ACRONYMS

MCF	Multichannel Filter
TFO	Tonto Forest Seismological Observatory

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SECTION I
INTRODUCTION

In applying linear least-mean-square-error theory to a single-channel prediction problem, a reference channel is estimated by linearly combining a group of channels $\underline{y}$ so that the mean-square-error is a minimum. The minimum-power array theory is a generalization of this concept where the reference channel itself is a linear combination of another group of channels $\underline{x}$ and has unity power.

In this report, vectors $\underline{x}$ and $\underline{y}$ represent two ordered sets of complex Fourier transforms obtained at a given frequency from the two groups of seismometer outputs. There are now two sets of linear multichannel filters (MCF's): one operates on group $\underline{x}$ to generate the reference channel; the other operates on group $\underline{y}$ to predict the reference channel.

Minimization of the mean-square-error by varying the two sets of MCF's leads to the solution of a generalized eigenvalue matrix equation. The minimizing pair of multichannel filters are known as the group-coherence filters. The multichannel filter formed from the difference between the two group-coherence filters is called the minimum-power array processor.

Mean-square-error is numerically equal to the fraction of the power in the normalized reference channel (the channel obtained by applying the group-coherence filter to the set of channels $\underline{x}$) which cannot be linearly predicted from group $\underline{y}$. The numerical value of the group coherence is defined to be the predictable fraction of the power in the reference channel; thus, a minimum mean-square-error is associated with a maximum group coherence, and vice versa.

An important property of group coherence is its invariance with respect to any nonsingular linear transformation of the channels within the two groups; i.e., group coherence is unchanged by scaling, by frequency filtering, or by combining the channels within a group by any linear reversible network filter. For seismometer arrays, the group coherence between two arrays is independent of any inequalization problems possessed by the seismometers in the arrays; however, if seismometer inequalization is severe, the wavenumber responses of the group-coherence filters differ from reasonable wavenumber power responses. Except for this inequalization effect, the wavenumber responses of the two group-coherence filters should tend to peak and be highly similar in regions where the wavenumber power spectrum of the array data is a maximum. The wavenumber response of the minimum-power array processor, which is the difference between the two group-coherence filters, should have a small power response at the wavenumber peaks. Thus, highly coherent energy such as that generated by storms or earthquakes would appear as deep troughs in the wavenumber response of the minimum-power array processor. A reasonable estimate of the wavenumber power spectrum should be possible from the wavenumber response of the minimum-power array processor.

If the weights of the group-coherence filters were adjusted to compensate for seismometer inequalizations, the filters' wavenumber responses would agree with the noise wavenumber power spectrum. Adjustments could be made by fixing the filter weight for one seismometer (the center one, for example) as unity and varying the filter weights corresponding to other seismometers so that the difference between the group-coherence filter wavenumber responses and the estimated noise wavenumber spectrum would be a minimum. The possibility that this processing scheme would lead to determining the amount of seismometer inequalization motivated the investigation covered in this report.

The broader objectives of this study are

- To investigate whether the wavenumber response of the minimum-power array processor can be used for estimating the noise wavenumber power spectrum by detecting and isolating regions of highly coherent energy

- To investigate whether the group-coherence filters and their wavenumber power responses can be used for determining the amplitude and phase-response inequalizations of the seismometers in the array

Data used in the study are from the TFO long-noise sample which was the subject of Array Research Special Report No. 23.[1] Also used are synthetic data modeled to resemble the TFO data but having no seismometer equalization problem. The following results concerning group coherence have been derived from this research.

The group-coherence technique is excellent for measuring the basic similarity between the two arrays of seismometers, and the group-coherence filters are useful if the objective is to generate the maximum coherent MCF outputs. The group-coherence concept is of little value in determining seismometer inequalization — at least, between array groups having small array separation, such as TFO. This is because the wavenumber response of the minimum-power filter lacks the resolution needed for estimating the noise wavenumber power spectrum.

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SECTION II

MINIMUM-POWER ARRAY PROCESSING THEORY

A. GENERALIZED MULTICHANNEL PREDICTION

The term "channel" in this report denotes a complex random variable. The Fourier transform of the output of a seismometer, since it is a complex random variable, is called a seismic channel.

Let an array of m + n seismic channels (Figure II-1) be partitioned into two ordered groups $\underline{x} = \{x_i \mid i = 1, \ldots, m\}$ and $\underline{y} = \{y_j \mid j = 1, \ldots n\}$ where $\underline{x}$ and $\underline{y}$ form two complex multivariate random column vectors. In the generalized multichannel prediction problem, group $\underline{x}$ is linearly combined to form a reference channel of unity power, and a linear combination of group $\underline{y}$ is used for obtaining a least-mean-square-error estimate of the normalized reference channel.

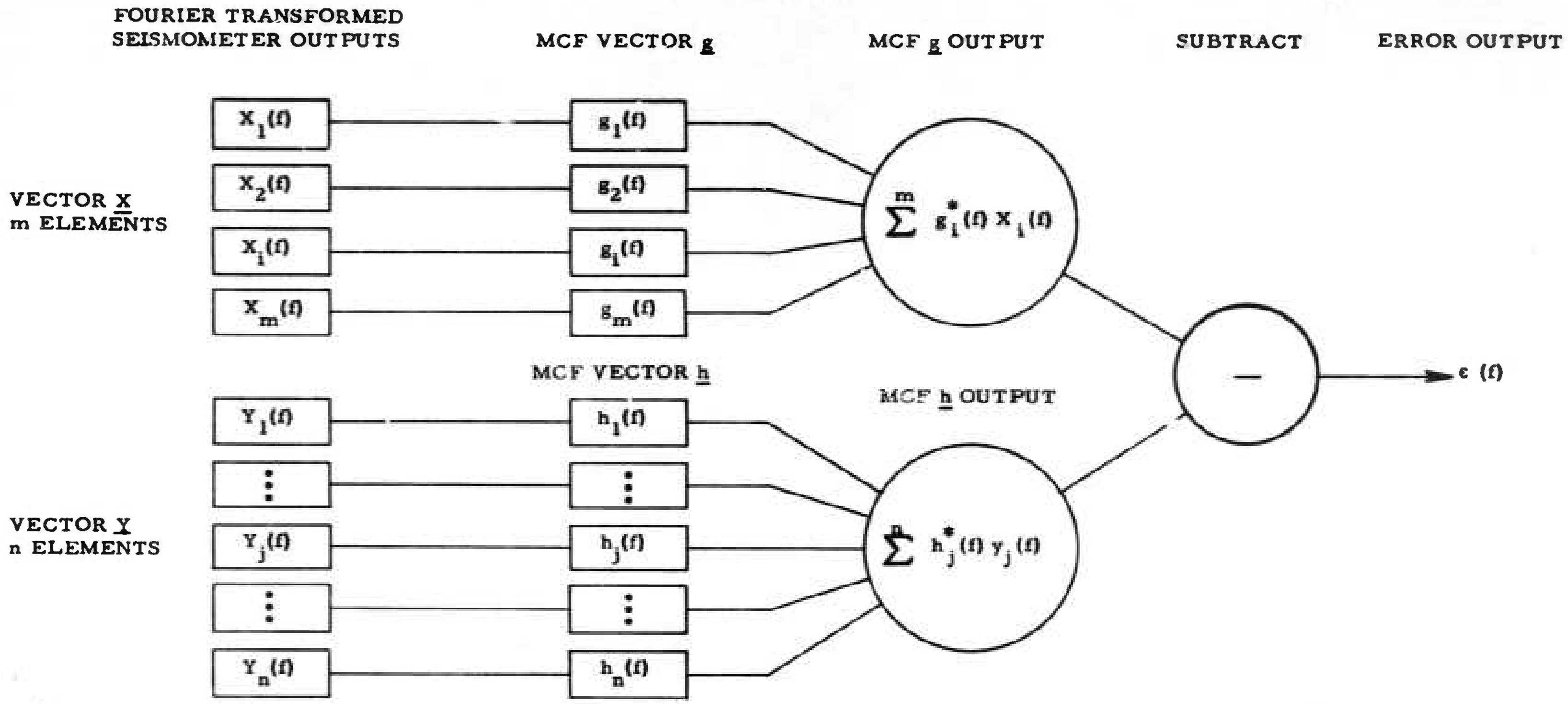
Let the two arbitrary sets of complex linear multichannel filters which operate on groups $\underline{x}$ and $\underline{y}$ be $\underline{g} = \{g_i \mid i = 1, \ldots, m\}$ and $\underline{h} = \{h_j \mid j = 1, \ldots, n\}$, respectively, and let ϵ be the difference between the outputs of $\underline{g}$ and $\underline{h}$. Note that MCF $\underline{g}$ generates the reference channel, that MCF $\underline{h}$ is the prediction filter, and that ϵ is a complex random variable which, in the future, will be referred to as error. Thus,

$$\underline{g}^T \underline{x} - h^T \underline{y} = \epsilon$$

or

$$\begin{pmatrix} \underline{g} \\ -\underline{h} \end{pmatrix}^T \begin{pmatrix} \underline{x} \\ \underline{y} \end{pmatrix} = \epsilon \qquad (2\text{-}1)$$

where T is used to denote conjugate transpose.



Where

$\underline{X}$, $\underline{Y}$, $\underline{g}$, and $\underline{h}$ are complex vectors and the asterisk is the complex conjugate.

Problem:

Find $\underline{g}$ and $\underline{h}$ such that the mean-square error $\overline{|\epsilon|}^2$ is a minimum and that the power output of the MCF $\underline{g}$ is unity. If $\underline{g}_k$ and $\underline{h}_k$ are such a pair, the MCF vector

$$\begin{Bmatrix} \underline{g}_k \\ -\underline{h}_k \end{Bmatrix}$$

is known as the minimum power array processor.

Figure II-1. Generalized Multichannel Prediction at Fixed Frequency

B. MINIMIZATION OF MEAN-SQUARE-ERROR

The mean-square-error is given by

$$\overline{|\epsilon|^2} = \overline{\epsilon\epsilon^T} = \overline{\left(\begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}^T \begin{Bmatrix} \underline{x} \\ \underline{y} \end{Bmatrix}\right) \left(\begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}^T \begin{Bmatrix} \underline{x} \\ \underline{y} \end{Bmatrix}\right)^T}$$

where a line indicates the average value. Therefore,

$$\overline{|\epsilon|^2} = \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}^T \overline{\begin{Bmatrix} \underline{x} \\ \underline{y} \end{Bmatrix} \begin{Bmatrix} \underline{x} \\ \underline{y} \end{Bmatrix}^T} \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}$$

$$= \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix} \tag{2-2}$$

where

$\underline{\underline{\Omega}}_{xx}$ is autopower matrix for group $\underline{x}$

$\underline{\underline{\Omega}}_{yy}$ is autopower matrix for group $\underline{y}$

$\underline{\underline{\Omega}}_{xy}$ is crosspower matrix for groups $\underline{x}$ and $\underline{y}$

$\underline{\underline{\Omega}}_{yx} = \underline{\underline{\Omega}}_{xy}^T$

It is desired to find $\underline{g}$ and $\underline{h}$, which will minimize $\overline{|\epsilon|^2}$, under the constraint that $\overline{|g^T \underline{x}|^2}$ is unity.

Keeping $\underline{g}$ fixed, the variation of $\overline{|\epsilon|^2}$ with respect to $\underline{h}$ in Equation 2-2 gives

$$\delta\,\overline{|\epsilon|^2} = \begin{Bmatrix} \underline{0} \\ -\delta\underline{h} \end{Bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix} + \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{Bmatrix} \underline{0} \\ -\delta\underline{h} \end{Bmatrix} \tag{2-3}$$

where $\underline{0}$ is the null vector and δ denotes the variation. Note that

$$\left(\begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{Bmatrix} \underline{0} \\ -\delta\underline{h} \end{Bmatrix}\right)^T = \begin{Bmatrix} \underline{0} \\ -\delta\underline{h} \end{Bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{Bmatrix} \underline{g} \\ -\underline{h} \end{Bmatrix}$$

$$= -\delta\underline{h}^T \left(\underline{\underline{\Omega}}_{yx}\, \underline{g} - \underline{\underline{\Omega}}_{yy}\, \underline{h}\right) \qquad (2\text{-}4)$$

From Equations 2-3 and 2-4, it is seen that

$$\delta\, \overline{|\epsilon|^2} = -2 \text{ Real } \left[\delta\underline{h}^T \left(\underline{\underline{\Omega}}_{yx}\, \underline{g} - \underline{\underline{\Omega}}_{yy}\, \underline{h}\right)\right] \qquad (2\text{-}5)$$

The condition for $\overline{|\epsilon|^2}$ to be stationary is that, for an infinitesimal change $\delta\underline{h}^T$, $\delta\, \overline{|\epsilon|^2} = 0$. This implies that the coefficient of $\delta\underline{h}^T$ in Equation 2-5 must vanish; i.e.,

$$\underline{\underline{\Omega}}_{yx}\, \underline{g} - \underline{\underline{\Omega}}_{yy}\, \underline{h} = 0$$

or

$$\underline{\underline{\Omega}}_{yy}\, \underline{h} = \underline{\underline{\Omega}}_{yx}\, \underline{g} \qquad (2\text{-}6)$$

Since $\underline{\underline{\Omega}}_{yy}$ is a covariance matrix, it is positive definite and, hence, nonsingular; i.e., Ω_{yy}^{-1} exists. Premultiplying both sides of Equation 2-6 gives

$$\underline{h} = \underline{\underline{\Omega}}_{yy}^{-1}\, \underline{\underline{\Omega}}_{yx}\, \underline{g} \qquad (2\text{-}7)$$

Substituting $\underline{h}$ from Equation 2-7 into Equation 2-2 and factoring gives

$$\left\{\underline{g}\right\}^T \begin{bmatrix} \underline{\underline{I}} \\ \hdashline -\underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx} \end{bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{bmatrix} \underline{\underline{I}} \\ \hdashline -\underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx} \end{bmatrix} \left\{\underline{g}\right\}$$

$$= \left\{\underline{g}\right\}^T \left[\left(\underline{\underline{\Omega}}_{xx} - \underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}\right) \quad \underline{\underline{0}}\right] \begin{bmatrix} \underline{\underline{I}} \\ \hdashline -\underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx} \end{bmatrix} \left\{\underline{g}\right\} = \overline{|\epsilon|^2}$$

or

$$\underline{g}^T \left(\underline{\underline{\Omega}}_{xx} - \underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}\right) \underline{g} = \overline{|\epsilon|^2} \tag{2-8}$$

where $\underline{\underline{I}}$ is the m x m identity matrix and $\underline{\underline{0}}$ is the m x n null matrix.

From Equation 2-8, it is seen that mean-square-error $\overline{|\epsilon|^2}$ depends only on the choice of the MCF vector $\underline{g}$ which generates the reference channel. The problem now is to find a minimizing MCF vector $\underline{g}$ which is normalized to unity output power. Since the matrix $\underline{\underline{\Omega}}_{yy}$ is positive definite and since $\underline{\underline{\Omega}}_{yx} = \underline{\underline{\Omega}}_{xy}^T$, the matrix $\underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}$ is positive definite. The matrix $\underline{\underline{\Omega}}_{xx}$, because it is a covariance matrix, also is positive definite. From matrix theory,[2] if either of the two matrices $\underline{\underline{\Omega}}_{xx}$ or $\underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}$ is positive definite, there exists a nonsingular matrix which will simultaneously reduce both the $\underline{\underline{\Omega}}_{xx}$ and $\underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}$ matrices to a diagonal form. Let $\underline{\underline{G}}$ be such an m x m matrix normalized so that

$$\underline{\underline{G}}^T \underline{\underline{\Omega}}_{xx} \underline{\underline{G}} = \underline{\underline{I}} \tag{2-9}$$

and

$$\underline{\underline{G}}^T \left[\underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}\right] \underline{\underline{G}} = |\underline{\underline{\lambda}}|^2 \qquad (2\text{-}10)$$

where $|\underline{\underline{\lambda}}|^2$ is a diagonal m x m matrix.

Equations 2-9 and 2-10 are generalized eigenvalue matrix equations.

Since $\underline{\underline{G}}$ is an m x m nonsingular matrix, its column vectors form a basis for generating arbitrary MCF's for group $\underline{x}$. Let $\underline{g}$ be such an MCF given by

$$\underline{g} = \underline{\underline{G}}\, \underline{c}$$

where $\underline{c} = \{c_i \mid i = 1, \ldots, m\}$ is a set of arbitrary complex scalars. Normalizing $\underline{g}$ to unity output power gives

$$\underline{g}^T \underline{\underline{\Omega}}_{xx} \underline{g} = 1 \qquad (2\text{-}11)$$

or

$$\underline{c}^T \underline{\underline{G}}^T \underline{\underline{\Omega}}_{xx} \underline{\underline{G}}\, \underline{c} = 1 \qquad (2\text{-}12)$$

Since, from Equation 2-9, $\underline{\underline{G}}^T \underline{\underline{\Omega}}_{xx} \underline{\underline{G}} = \underline{\underline{I}}$, Equation 2-12 simplifies to

$$\underline{c}^T \underline{c} = 1$$

or

$$\sum^{m} c_i^2 = 1 \qquad (2\text{-}13)$$

Equation 2-13 gives the condition for unity normalization.

Pre- and postmultiplying $\underline{\underline{\Omega}}_{xy}\,\underline{\underline{\Omega}}_{yy}^{-1}\,\underline{\underline{\Omega}}_{yx}$ by $\underline{g}^T$ and $\underline{g}$, respectively, yields

$$\underline{g}^T\,\underline{\underline{\Omega}}_{xy}\,\underline{\underline{\Omega}}_{yy}^{-1}\,\underline{\underline{\Omega}}_{yx}\,\underline{g} = \underline{c}^T\,\underline{\underline{G}}^T\,\underline{\underline{\Omega}}_{xy}\,\underline{\underline{\Omega}}_{yy}^{-1}\,\underline{\underline{\Omega}}_{yx}\,\underline{\underline{G}}\,\underline{c} \tag{2-14}$$

Since, from Equation 2-10, $\underline{\underline{G}}^T\,\underline{\underline{\Omega}}_{xy}\,\underline{\underline{\Omega}}_{yy}^{-1}\,\underline{\underline{\Omega}}_{yx}\,\underline{\underline{G}} = |\underline{\underline{\lambda}}|^2$, Equation 2-14 reduces to

$$\underline{g}^T\,\underline{\underline{\Omega}}_{xy}\,\underline{\underline{\Omega}}_{yy}^{-1}\,\underline{\underline{\Omega}}_{yx}\,\underline{g} = \underline{c}^T\,|\underline{\underline{\lambda}}|^2\,\underline{c} = \sum^{m} |c_i|^2\,|\lambda_i|^2 \tag{2-15}$$

Subtracting Equation 2-15 from Equation 2-11 and factoring,

$$\underline{g}^T\left(\underline{\underline{\Omega}}_{xx} - \underline{\underline{\Omega}}_{xy}\,\underline{\underline{\Omega}}_{yy}^{-1}\,\underline{\underline{\Omega}}_{yx}\right)\underline{g} = 1 - \sum^{m} |c_i|^2\,|\lambda_i|^2 \tag{2-16}$$

Comparing Equations 2-8 and 2-16, it can be seen that

$$\overline{|\epsilon|^2} = 1 - \sum^{m} |c_i|^2\,|\lambda_i|^2 \tag{2-17}$$

Equation 2-17 gives the mean-square-error associated with an arbitrary normalized filter $\underline{g} = \underline{\underline{G}}\,\underline{c}$. $\overline{|\epsilon|^2}$ is minimized by setting $c_i = 1$ when $|\lambda_i|^2 = |\lambda_{max}|^2$ and setting all other c_i's $= 0$ in Equation 2-17; i.e.,

$$\overline{|\epsilon_{min}|^2} = 1 - |\lambda_{max}|^2 \tag{2-18}$$

C. GROUP COHERENCE

Section I states that group coherence is measured by the maximum predictable fraction of power in the normalized reference channel. Since the reference channel is normalized to unity power and since $|\epsilon_{min}|^2$ is the minimum prediction error, Equation 2-18 shows that the group coherence equals the largest eigenvalue $|\lambda_{max}|^2$.

The eigenvector associated with the largest eigenvalue is obtained by solving Equations 2-9 and 2-10, using an iterative scheme called the power method. (This scheme has been described in Large-Array Signal and Noise Analysis Special Scientific Report No. 13.)[3] If $\underline{g}_{max}$ is such an eigenvector, then, from Equations 2-9 and 2-10,

$$\underline{g}_{max}^{T} \, \underline{\underline{\Omega}}_{xx} \, \underline{g}_{max} = 1 \tag{2-19}$$

and

$$\underline{g}_{max}^{T} \left[\underline{\underline{\Omega}}_{xy} \, \underline{\underline{\Omega}}_{yy}^{-1} \, \underline{\underline{\Omega}}_{yx} \right] \underline{g}_{max} = \left| \lambda_{max} \right|^{2} \tag{2-20}$$

The eigenvector $\underline{g}_{max}$ is known as the group-coherence MCF associated with group $\underline{x}$.

The group-coherence MCF $\underline{h}_{max}$ associated with group $\underline{y}$ is obtained by substituting $\underline{g}_{max}$ into Equation 2-7:

$$\underline{h}_{max} = \underline{\underline{\Omega}}_{yy}^{-1} \, \underline{\underline{\Omega}}_{yx} \, \underline{g}_{max} \tag{2-21}$$

The minimum-power array processor output is obtained by replacing $\underline{g}$ and $\underline{h}$ in Equation 2-1 by $\underline{g}_{max}$ and $\underline{h}_{max}$, respectively.

D. AUTO- AND CROSSPOWER SPECTRA OF THE TWO MAXIMUM-COHERENT CHANNELS

The autopower spectrum of the reference channel is given by Equation 2-19. To obtain the autopower spectrum of the estimated reference channel, take the conjugate transpose of Equation 2-21 while recalling that $\underline{\underline{\Omega}}_{yy}^{-1}$ is Hermitian and $\underline{\underline{\Omega}}_{yx}^{T} = \underline{\underline{\Omega}}_{xy}$. Thus,

$$\underline{h}_{max}^{T} = \underline{g}_{max}^{T} \underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \tag{2-22}$$

By postmultiplying Equation 2-18 on both sides by $\underline{\underline{\Omega}}_{yy} \underline{h}_{max}$ and noting that $\underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yy} = \underline{\underline{I}}$, where $\underline{\underline{I}}'$ is an n x n identity matrix,

$$\underline{h}_{max}^{T} \underline{\underline{\Omega}}_{yy} \underline{h}_{max} = \underline{g}_{max}^{T} \underline{\underline{\Omega}}_{xy} \underline{h}_{max} \tag{2-23}$$

By replacing $\underline{h}_{max}$ on the right-hand side of Equation 2-23 by its equivalent expression from Equation 2-21,

$$\begin{aligned} \underline{h}_{max}^{T} \underline{\underline{\Omega}}_{yy} \underline{h}_{max} &= \underline{g}_{max}^{T} \underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx} \underline{g}_{max} \\ &= \underline{g}_{max}^{T} \left(\underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}\right) \underline{g}_{max} \end{aligned} \tag{2-24}$$

A comparison of Equations 2-24 and 2-20 shows that

$$\underline{h}_{max}^{T} \underline{\underline{\Omega}}_{yy} \underline{h}_{max} = |\lambda_{max}|^{2} \tag{2-25}$$

The expression on the left-hand side of Equation 2-25 is the autopower spectrum of the estimated reference channel, which is automatically normalized to the numerical value of the group coherence $\left(|\lambda_{max}|^{2}\right)$.

A comparison of Equations 2-23 and 2-25 shows that

$$\underline{g}_{max}^{T} \underline{\underline{\Omega}}_{xy} \underline{h}_{max} = |\lambda_{max}|^2 \qquad (2\text{-}26)$$

The expression on the left-hand side of Equation 2-26 is the crosspower spectrum between the reference channel and the estimated reference channel.

E. SUMMARY OF MINIMUM-POWER ARRAY PROCESSING

The salient features of this section are summarized below:

- Equations 2-9 and 2-10 can be iteratively solved, using the power method, to give the group coherence $|\lambda_{max}|^2$ and the MCF $\underline{g}_{max}$; the MCF $\underline{g}_{max}$ generates the maximum-coherent reference channel from group $\underline{x}$
- MCF $\underline{h}_{max}$, which estimates the reference channel by linearly operating on group $\underline{y}$, can be obtained from $\underline{g}_{max}$ by Equation 2-7
- The minimum-power array processor is the MCF

$$\begin{pmatrix} \underline{h}_{max} \\ -\underline{g}_{max} \end{pmatrix}$$

and the power in the corresponding error channel is given by

$$\overline{|\epsilon_{min}|^2} = 1 - |\lambda_{max}|^2$$

F. THE COMPLETE SET OF m EIGENVALUES

Thus far, only the group coherence which is numerically equal to the largest eigenvalue of Equations 2-9 and 2-10 has been discussed. However, the generalized eigenvalue equations have m-1 other solutions.

The properties of the entire set of m eigenvalues and the associated eigenvectors are investigated in Appendix A.

SECTION III
DATA PREPARATION

The minimum-power array processing technique consists of the following three stages:

- Computation of the crosspower matrix at a chosen frequency
- Computation of the maximum group-coherence MCF's for the two partitioned sets
- Computation of wavenumber responses of the two group-coherence MCF's and of the minimum-power array processor

Presented in this report are the results from two sets of models and the TFO long-noise sample.[1] The TFO crossarray (Figure III-1) was partitioned into four sets (Figure III-2) and group coherence was studied at various frequencies.

The three models designed at 0.22584 Hz, 0.52696 Hz, and 0.82808 Hz in model set 1 are close approximations to the TFO noise field. Figure III-3 shows these wavenumber spectrum models, using the following conventions.

A solid disk represents isotropic energy propagating above a certain minimum speed, a cylinder represents isotropic energy propagating at a fixed speed, and an arrow represents directional energy such as that generated by storms and earthquakes. Background mantle P-wave energy, due to its high apparent speed, is modeled by an 8-km/sec disk. The isotropic fundamental Rayleigh-wave energy is modeled by the 3-km/sec cylinder and higher-order modes by the 3-km/sec disk. Highly directional P waves and Rayleigh waves are represented by the arrows.

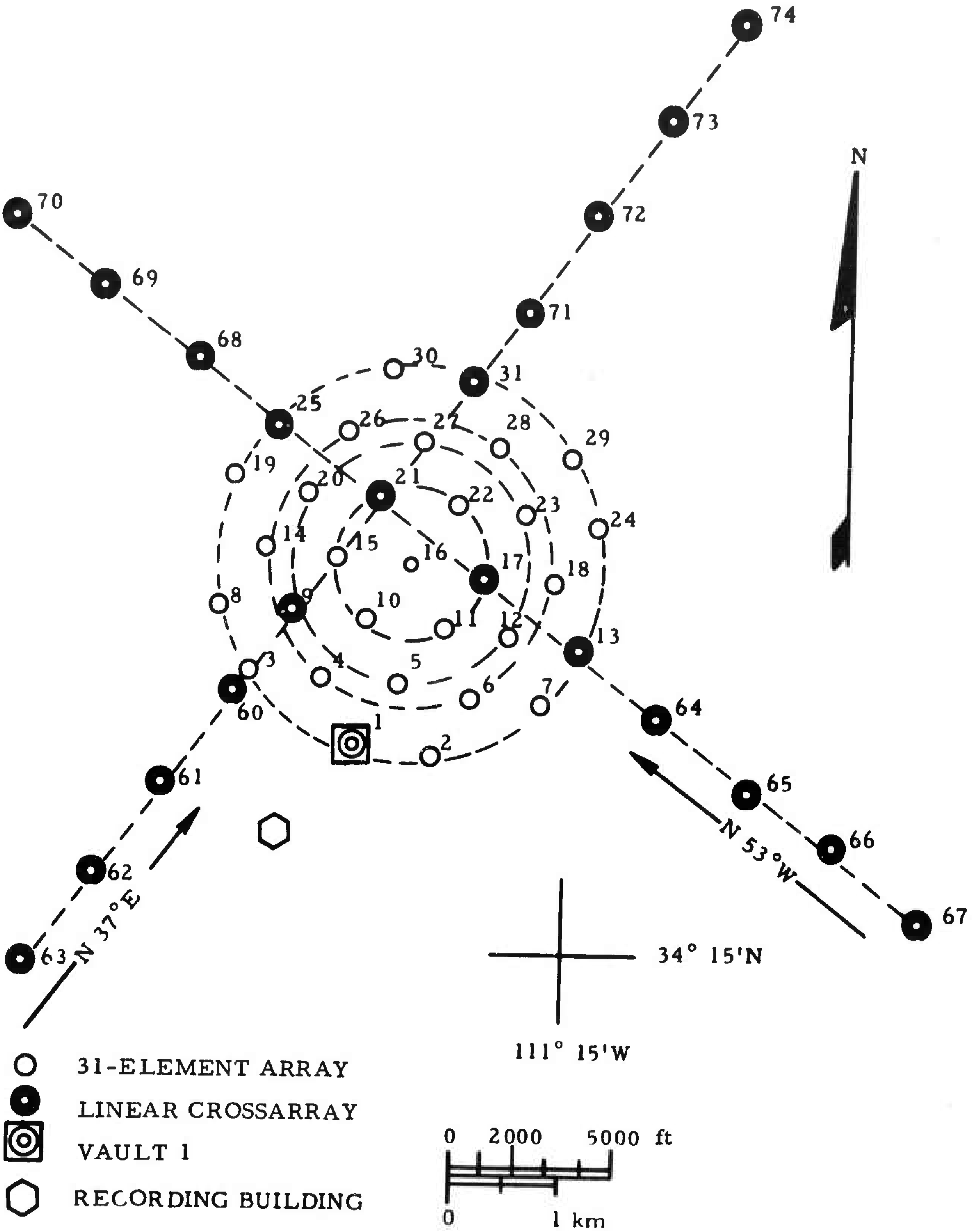


Figure III-1. Array at Tonto Forest Seismological Observatory (TFO)

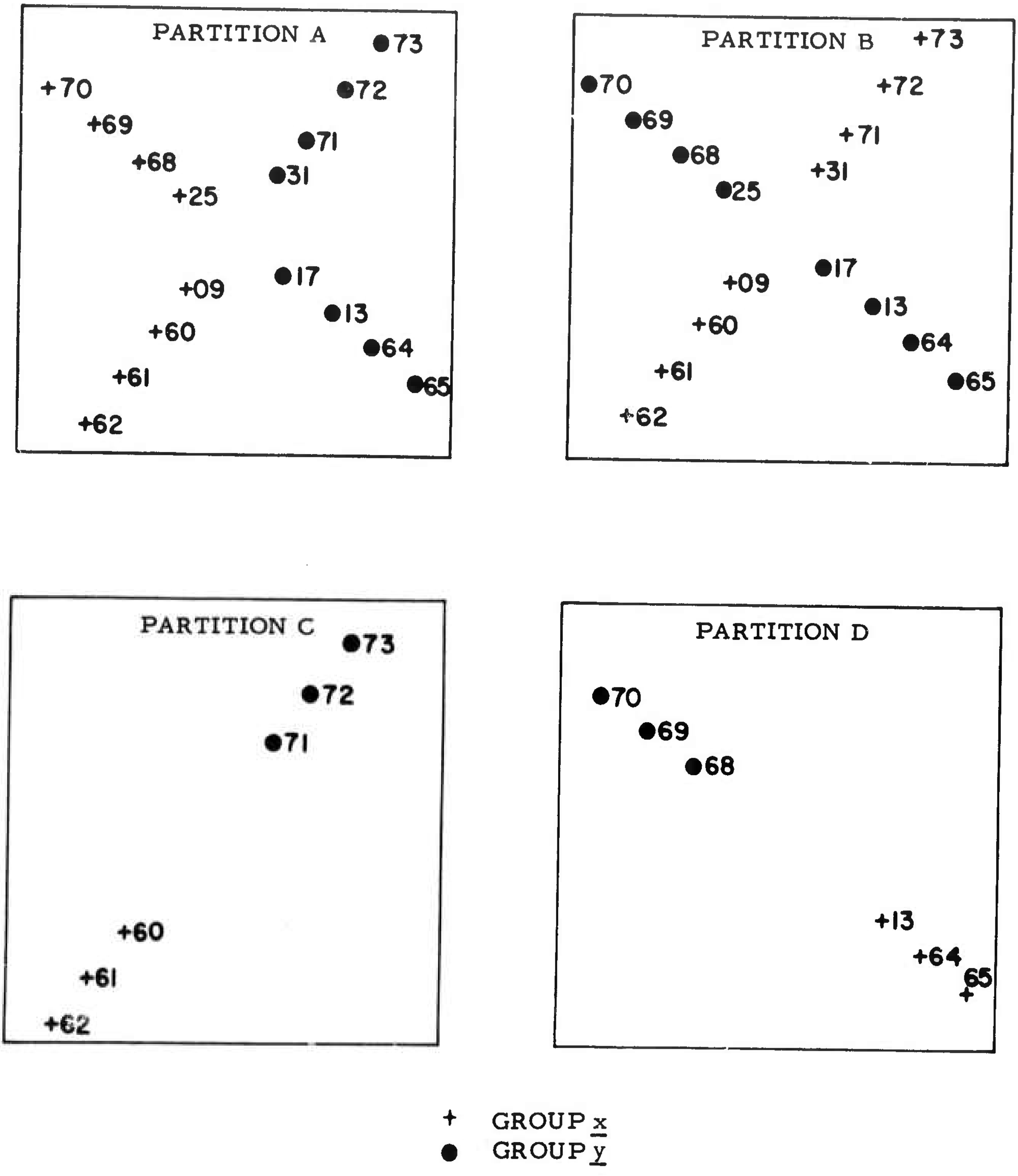


Figure III-2. Partitioned Sets of TFO Array

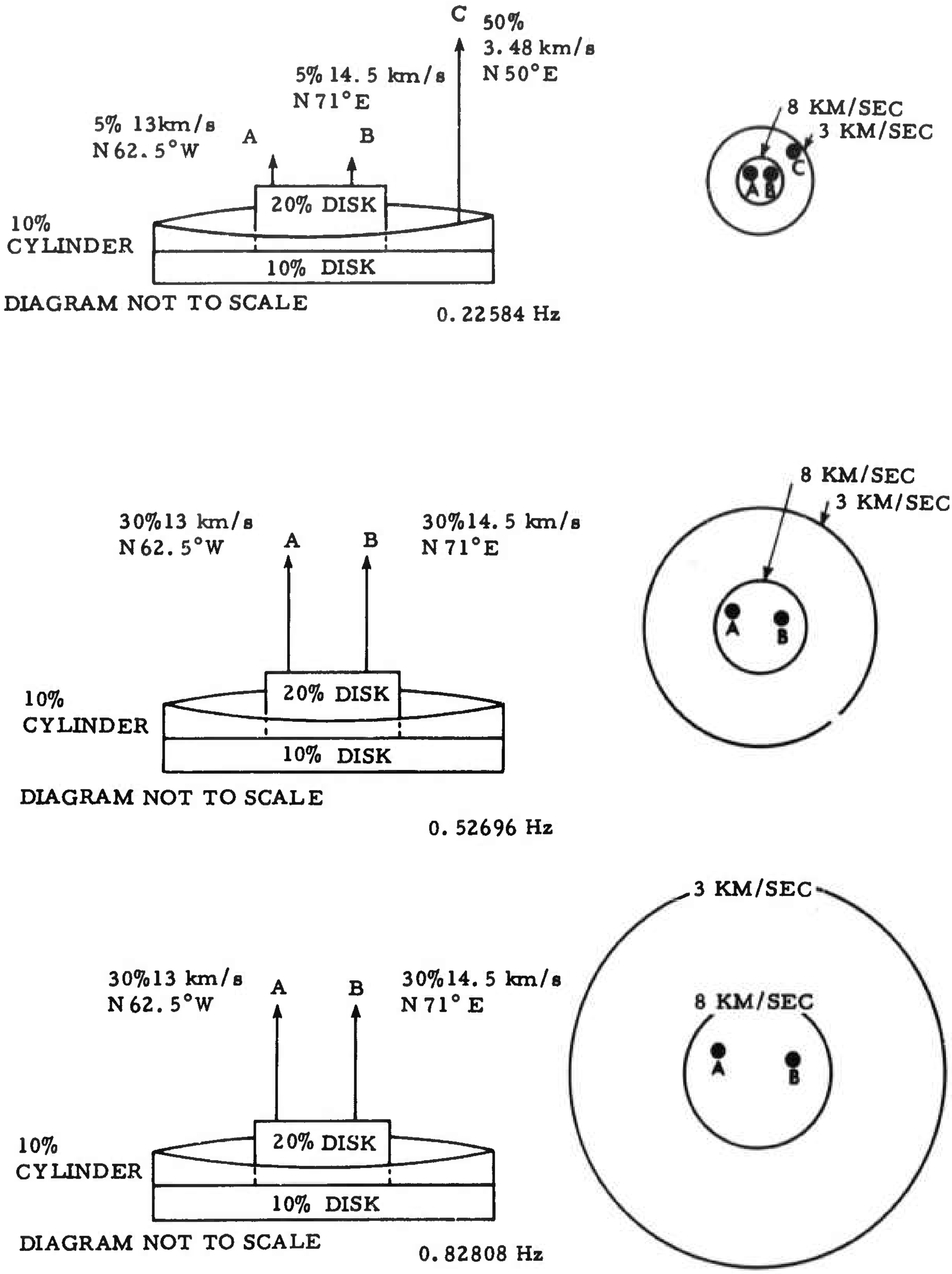


Figure III-3. TFO Noise Model Set 1 in Wavenumber Space

Model set 2 is the same as Model set 1 except that 1-percent white noise is added.

The crosspower matrices for these models are derived from formulas published in the final report on Seismometer Array and Data Processing Systems.[4] These formulas are reproduced in Appendix B.

The crosspower matrix for TFO data is computed from spectral estimates of the TFO noise sample[5] obtained by Bartlett-smoothing and Fourier-transforming the auto- and crosscorrelation functions discussed in an Array Research semiannual technical report.[6]

SECTION IV
DISCUSSION OF RESULTS

With respect to the original purpose of determining seismometer inequalization, the study of group coherence filters has been unsuccessful in achieving useful results. However, the following detailed discussion of results does illuminate the properties of group coherence filters and indicates that they will be useful for uncovering and analyzing coherent energy between subarrays.

A. PARTITIONS A AND B

Table IV-1 shows group coherences obtained from the four partitioned sets A, B, C, and D. Partitions A and B give high group coherences.

Minimum-power array MCF's computed from TFO data at 0.52696 Hz, corresponding to partitions A and B, are plotted as vectors in Figure IV-1. An arbitrary scale factor and an arbitrary phase reference are used for plotting these vectors. The scale factors are different for the two partitions.

Both vector diagrams show that the MCF's with the largest weights are located near the center seismometer, which is not included in these partitions. Thus, both groups $\underline{x}$ and $\underline{y}$ seem to be trying to predict the output of the missing center seismometer.

Figure IV-2 shows wavenumber responses of the minimum-power arrays (shown in Figure IV-1 as vector diagrams). These wavenumber responses are computed along the two TFO arms, i.e., S37°W-N37°E and S53°W-N53°E. Also shown in Figure IV-2 are the inverted K-line wavenumber power spectra,* which are projections of 2-dimensional power-density spectra onto the two arms of the TFO crossarray;[1] they serve as references for checking the performance of the minimum-power arrays.

---

*K-line wavenumber spectra are obtained from the maximum entropy spectral analysis, a technique developed by John P. Burg and presented in November 1967 at the 37th annual SEG meeting in Oklahoma City. The properties of K-line wavenumber spectra have been discussed and extensively illuminated in Array Research Special Report No. 23.[1]

Table IV-1

GROUP COHERENCES

Partitions	0.22584 Hz Model 1	0.22584 Hz Model 2	0.22584 Hz TFO Data	0.52696 Hz Model 1	0.52696 Hz Model 2	0.52696 Hz TFO Data	0.82808 Hz Model 1	0.82808 Hz Model 2	0.82808 Hz TFO Data
A	NC	0.980	0.980	NC	0.880	0.880	NC	0.750	0.82
B	NC	NC	NC	NC	NC	0.975	NC	NC	NC
C	0.995	0.880	0.860	0.600	0.400	0.270	0.378	0.360	0.390
D	0.999	0.840	0.860	0.560	0.300	0.320	0.358	0.340	0.320

NC: not computed

The broad low region in the wavenumber power responses of the minimum-power arrays suggests that a reasonable noise wavenumber power spectra should have a corresponding broad peak. This is not, in fact, the case, as can be seen by comparing these wavenumber responses with the inverted K-line wavenumber power spectra. However, a basic similarity between the two shapes can be seen, as both the minimum-power array wavenumber response and the inverted K-line wavenumber spectra indicate that the ambient seismic-noise energy sharply drops outside the 3-km/sec dashed lines.

B. PARTITIONS C AND D

Partitions C and D have been selected to provide higher resolution in the wavenumber response than achieved by partitions A and B. The minimum separation distance between the two groups of seismometers in partitions C and D is nearly 4 km, which is two times the minimum separation for partitions A and B. With increased separation, the responses of the two group-coherence filters can be similar only over a much narrower wavenumber region than in the cases of partitions A and B; also, as a result of increased separation between the groups, the coherences for partitions C and D are low (Table IV-1).

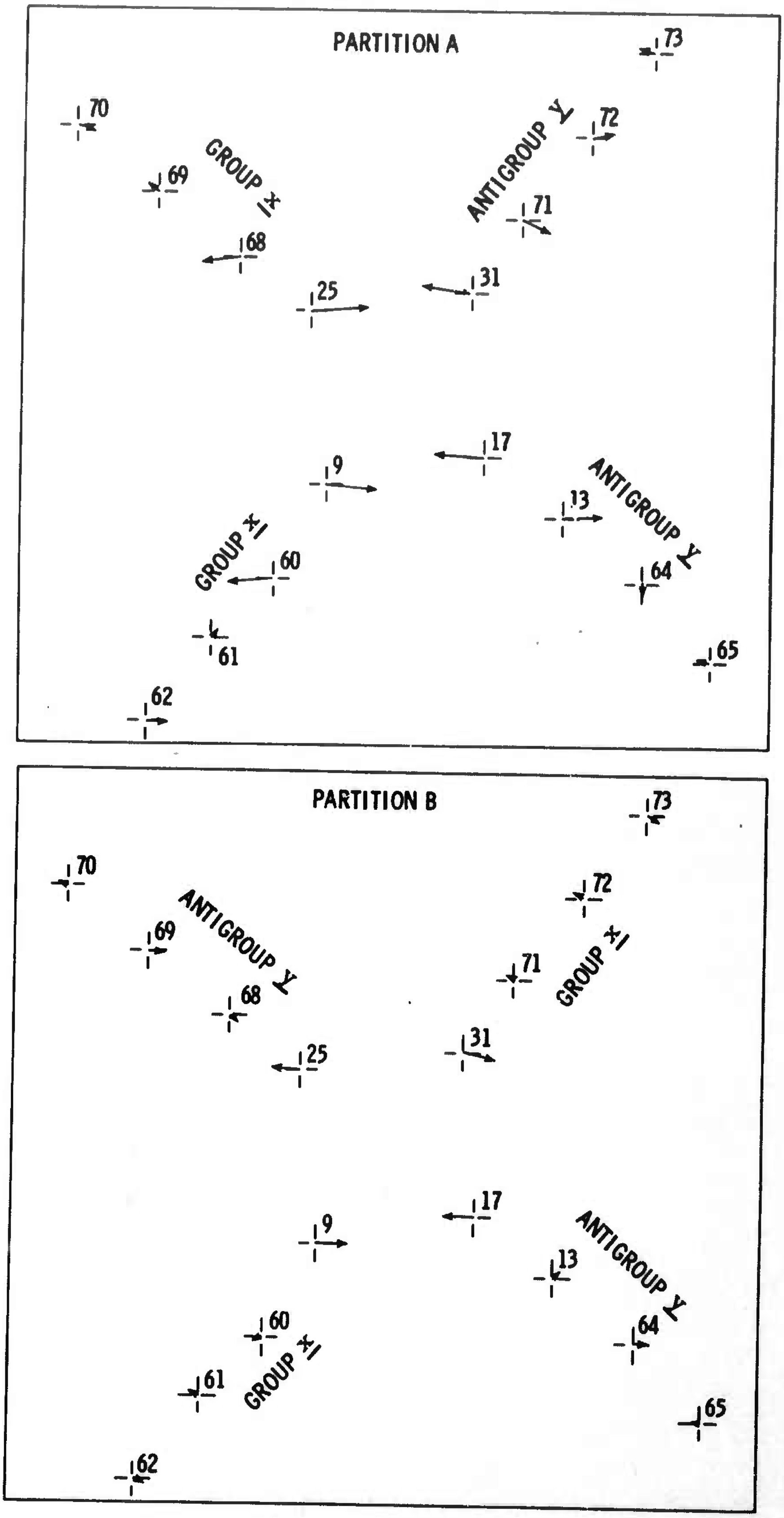


Figure IV-1. Vector Diagram of Minimum-Power Array Processor, TFO Noise Data: Frequency, 0.52696 Hz

Minimum-Power Array Wavenumber Response, Partition A

Minimum-Power Array Wavenumber Response, Partition B

Inverted K-Line Wavenumber Spectra

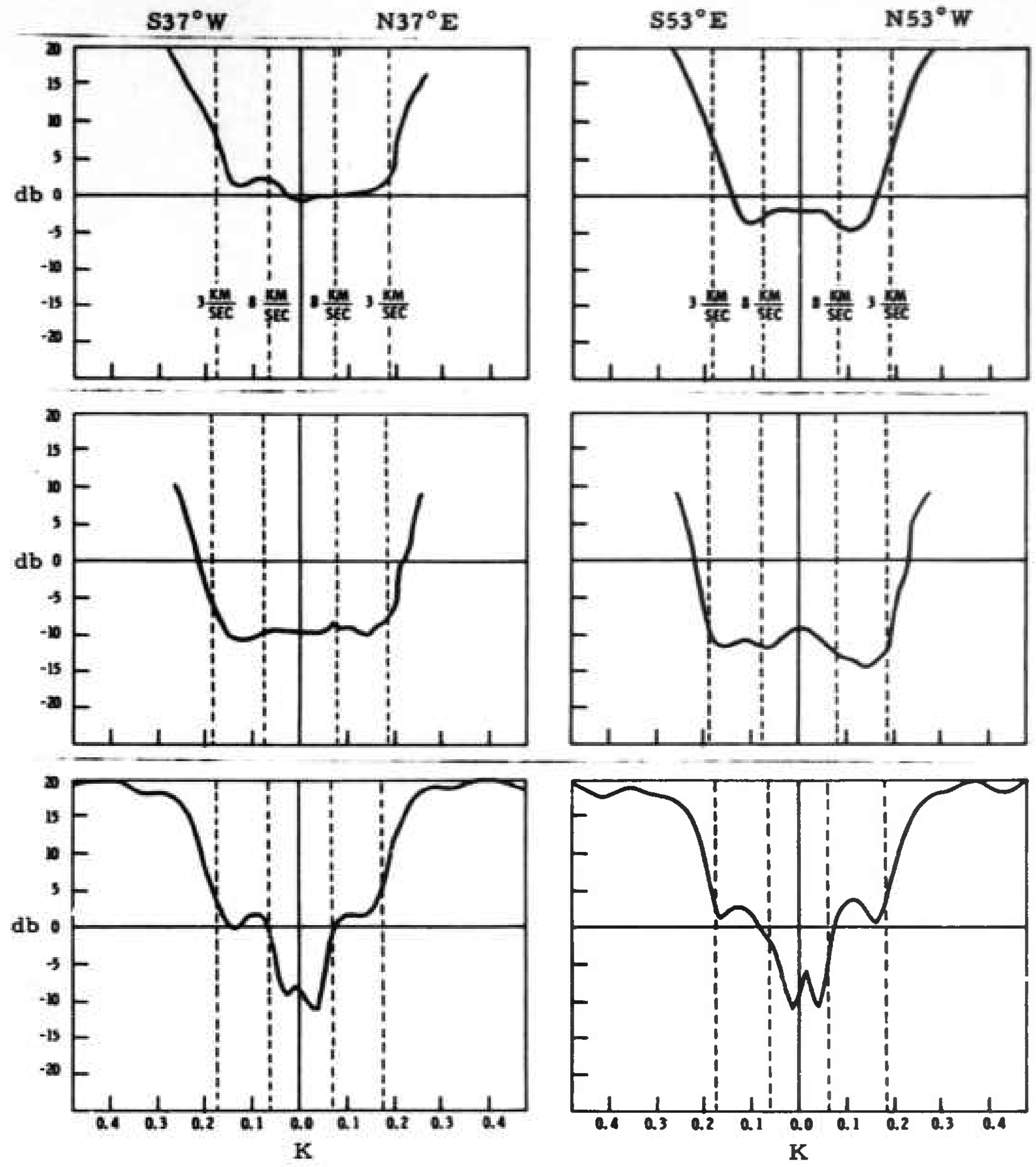


Figure IV-2. Minimum-Power Array Wavenumber Responses and Inverted K-Line Wavenumber Spectra, TFO Noise Data: Frequency, 0.52696 Hz

Figures IV-3 and IV-4 show the wavenumber power responses along the two TFO arms. Minimum-power array wavenumber responses in model set 1 generally resemble the energy distribution in the model but do not show the very strong directional Rayleigh wave coming from N50°E (0.22584 Hz). Minimum-power array wavenumber responses for model set 2 show poor resolution. Minimum-power array wavenumber responses obtained from TFO data generally resemble the corresponding inverted K-line wavenumber spectra obtained from the same data, but the former show poor resolution and are unreliable for estimating a reasonable wavenumber power spectra. This last conclusion is based on the presence of extraneous low regions in the wavenumber responses at 0.52696 Hz (TFO data, partition C). Since a similar effect is observed for synthetic data (model set 2, 0.52696 Hz, partition D), this phenomenon appears to be a property of the technique and not of inequalization.

C. 2-DIMENSIONAL WAVENUMBER POWER RESPONSE

Figure IV-5 shows 2-dimensional wavenumber responses of the minimum-power array computed at the three frequencies for partition D, model set 1.

Comparisons of these wavenumber responses with the wavenumber power distributions in the actual model (Figure III-1) again show that the technique lacks the resolution and reliability needed for estimating with any reasonable accuracy the wavenumber power spectra; at least, this is true for the array groups which have little separation, such as TFO.

D. GROUP-COHERENCE MULTICHANNEL FILTERS

The wavenumber responses of group-coherence MCF's show very poor resolution and, since no conclusions can be drawn from them, they are not presented in this report.

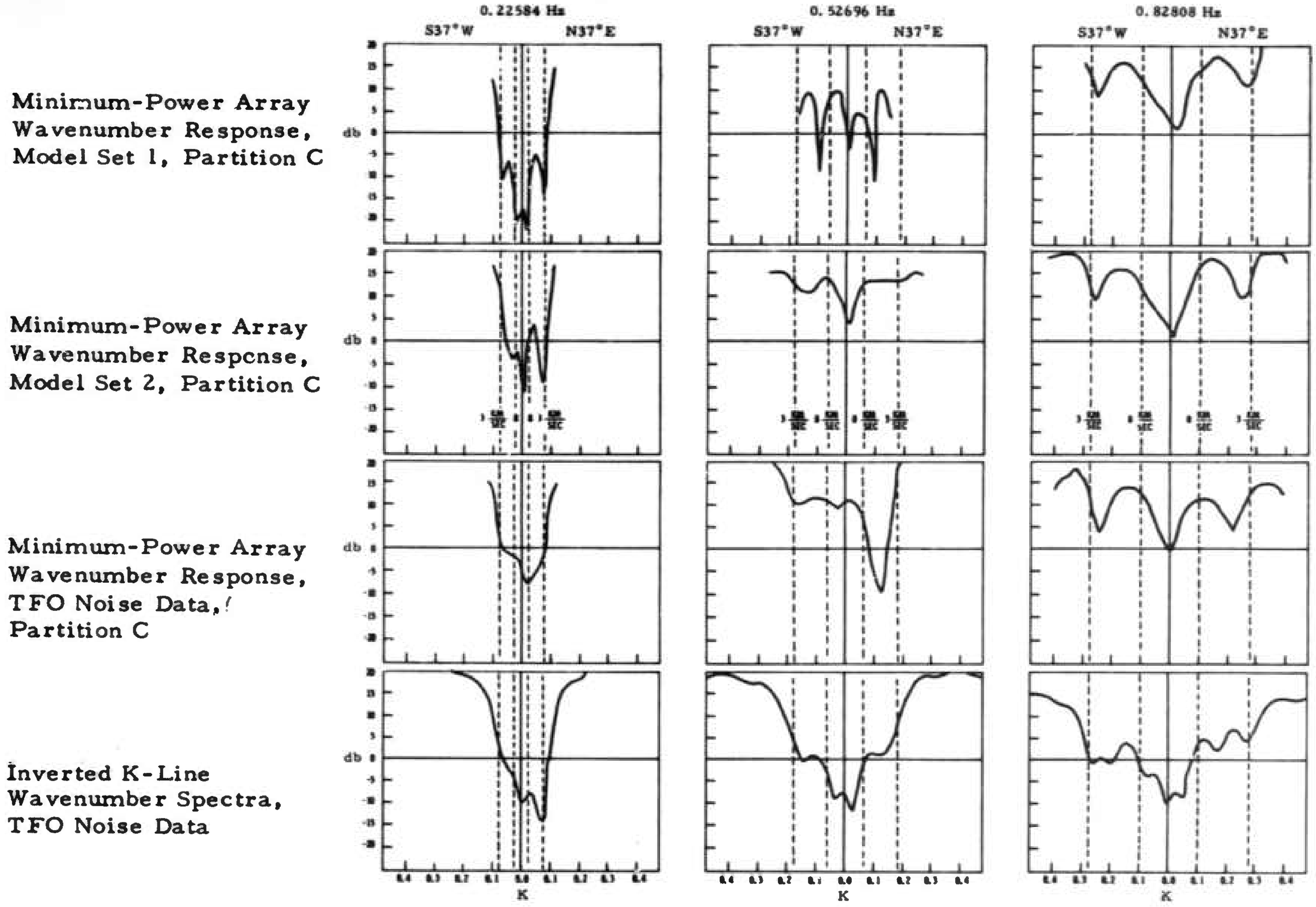


Figure IV-3. Comparisons of Minimum-Power Array Wavenumber Responses and Inverted K-Line Wavenumber Spectra, S37°W-N37°E Arm

Minimum-Power Array Wavenumber Response, Model Set 1, Partition D

Minimum-Power Array Wavenumber Response, Model Set 2, Partition D

Minimum-Power Array Wavenumber Response, TFO Noise Data, Partition D

Inverted K-Line Wavenumber Spectra, TFO Noise Data

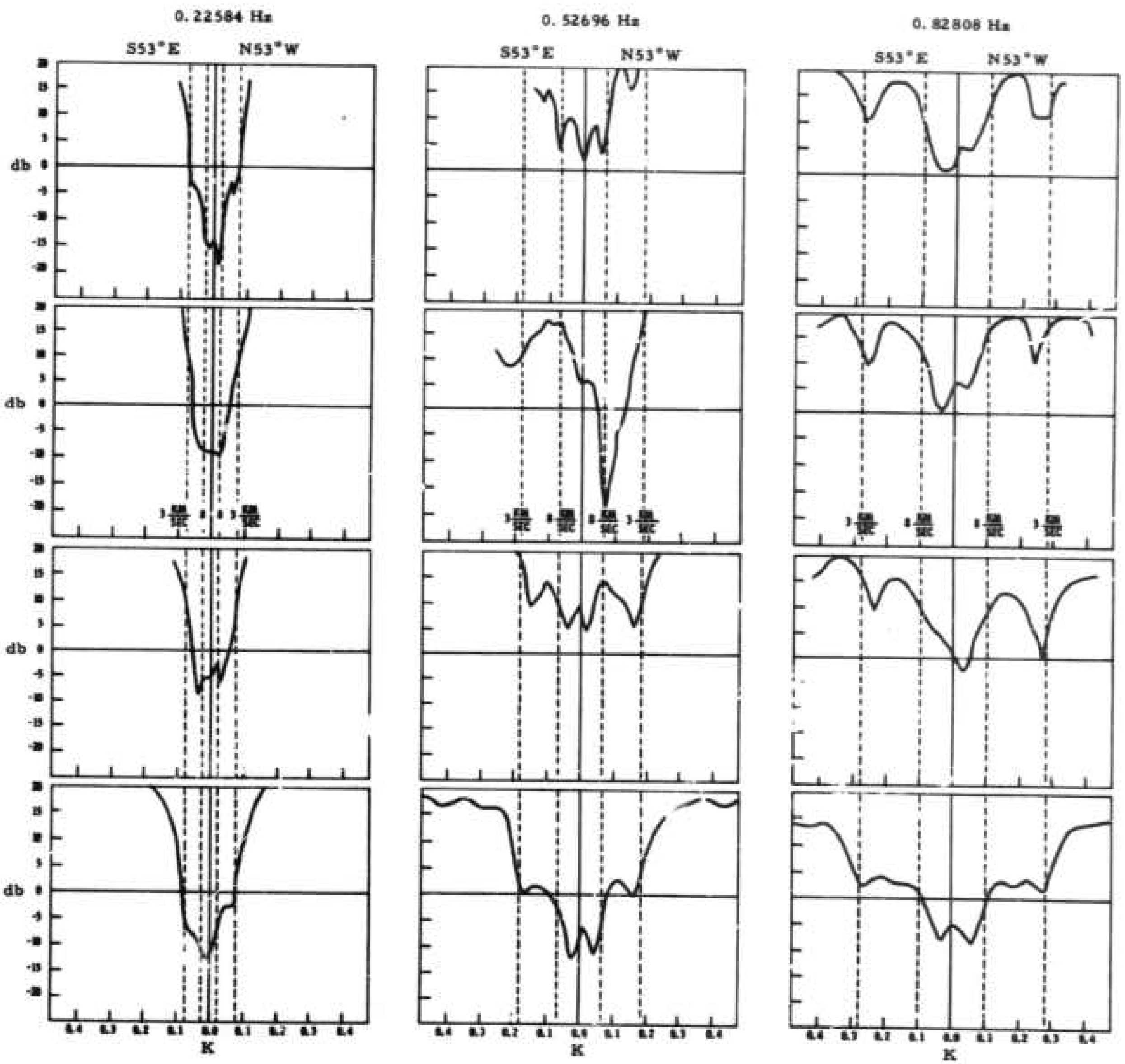


Figure IV-4. Comparisons of Minimum-Power Array Wavenumber Responses and Inverted K-Line Wavenumber Spectra, S53°E-N53°W Arm

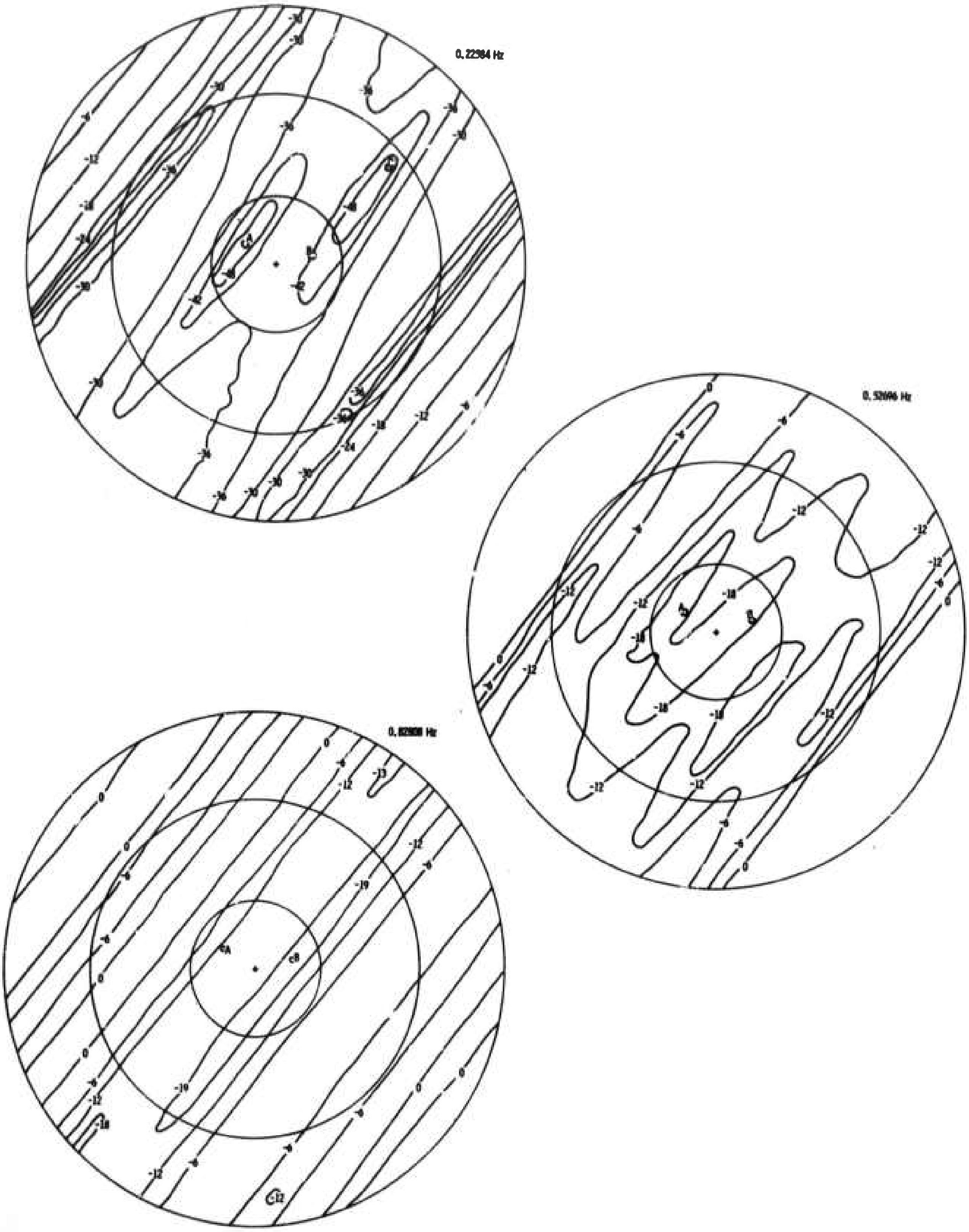


Figure IV-5. 2-Dimensional Wavenumber Response (in db) of Minimum-Power Array, Model Set 1, Partition D

SECTION V

REFERENCES

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APPENDIX A

THE m SOLUTIONS OF THE GENERALIZED EIGENVALUE PROBLEM

APPENDIX A

THE m SOLUTIONS OF THE GENERALIZED EIGENVALUE PROBLEM

The generalized eigenvalue problem is stated as

$$\underline{\underline{G}}^T \underline{\underline{\Omega}}_{xx} \underline{\underline{G}} = \underline{\underline{I}} \qquad \text{(A-1)}$$

and

$$\underline{\underline{G}}^T \left[\underline{\underline{\Omega}}_{xy} \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx}\right] \underline{\underline{G}} = |\underline{\underline{\lambda}}|^2 \qquad \text{(A-2)}$$

where

$\underline{\underline{I}}$ denotes m x m identity matrix

$|\underline{\underline{\lambda}}|^2$ denotes m x m diagonal matrix of eigenvalues

$\underline{\underline{G}}$ denotes m x m nonsingular matrix of eigenvectors

Section II of this report shows that a pair of MCF's $\underline{g}_{max}$ and $\underline{h}_{max}$ exist, corresponding to the eigenvalue $|\lambda_{max}|^2$, which produce two maximum coherent channels when linearly combined with groups $\underline{x}$ and $\underline{y}$, respectively. Similarly, a pair of MCF's $\underline{g}_i$ and $\underline{h}_i$ exist, corresponding to each of the m eigenvalues of the set

$$\left\{|\lambda_i|^2 \,\middle|\, i = 1, \ldots, m\right\}$$

where $\underline{g}_i$ is called the reference MCF and $\underline{h}_i$ is called the prediction MCF.

The MCF $\underline{h}_i$ is related to the MCF $\underline{g}_i$ by Equation 2-7, which is

$$\underline{h}_i = \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx} \underline{g}_i \qquad \text{(A-3)}$$

Since the set $\{\underline{g}_i \mid i = 1, \ldots, m\}$ is the m x m matrix $\underline{\underline{G}}$, the set $\underline{\underline{H}} = \{\underline{h}_i \mid i = 1, \ldots, m\}$ is an n x m matrix given by

$$\underline{\underline{H}} = \underline{\underline{\Omega}}_{yy}^{-1} \underline{\underline{\Omega}}_{yx} \underline{\underline{G}} \tag{A-4}$$

To complete the relationships between the matrices $\underline{\underline{G}}$, $\underline{\underline{H}}$, and $|\underline{\underline{\lambda}}|^2$, the following two equations are given:

$$\underline{\underline{H}}^T \underline{\underline{\Omega}}_{yy} \underline{\underline{H}} = |\underline{\underline{\lambda}}|^2 \tag{A-5}$$

and

$$\underline{\underline{G}}^T \underline{\underline{\Omega}}_{xy} \underline{\underline{H}} = |\underline{\underline{\lambda}}|^2 \tag{A-6}$$

These are equivalent to Equations 2-25 and 2-26 for the m eigenvalue case.

- Lemma 1

The m reference MCF's, which are the column vectors of the m x m matrix $\underline{\underline{G}}$, are linearly independent.

Since $\underline{\underline{G}}$ is nonsingular, its column vectors are linearly independent.

Q. E. D.

- Lemma 2

The m prediction MCF's which are the column vectors of the n x m matrix $\underline{\underline{H}}$ are linearly independent if the matrix $|\underline{\underline{\lambda}}|^2$ is nonsingular.

Rank is defined as the maximum number of linearly independent column or row vectors in a matrix. Since all the columns or rows of a nonsingular matrix are linearly independent, its rank equals the number of rows or columns. Thus, the rank of the m x m matrix $|\underline{\underline{\lambda}}|^2$ is m and that of the n x n matrix $\underline{\underline{\Omega}}_{yy}$ is n.

The ranks of the matrices on the two sides of Equation A-5 are given by

$$\rho\left(\underline{\underline{H}}^T\ \underline{\underline{\Omega}}_{yy}\ \underline{\underline{H}}\right) = \rho\left(|\underline{\underline{\lambda}}|^2\right) = m \tag{A-7}$$

where ρ denotes the rank. According to a theorem in matrix theory, the rank of a product of matrices cannot exceed the rank of any of the component matrices and, since $\rho(\underline{\underline{H}}) = \rho(\underline{\underline{H}}^T)$, Equation A-7 takes the form

$$\min\left[\rho(\underline{\underline{H}}),\ n\right] \geq m \tag{A-8}$$

Since $\underline{\underline{H}}$ is an n x m matrix, another theorem in matrix theory gives

$$\rho(\underline{\underline{H}}) \leq \min(m,\ n) \tag{A-9}$$

It is seen from Inequalities A-8 and A-9 that

$$\rho(\underline{\underline{H}}) = m \leq n \tag{A-10}$$

Equation A-10 says that the number of linearly independent vectors in $\underline{\underline{H}}$ is m and that this number cannot exceed the dimension (n) of the space.

Q. E. D.

- Lemma 3

The column vectors of the m x m matrix $\underline{\underline{G}}$, when used as reference MCF's on group $\underline{x}$, give m linearly independent outputs.

Letting the outputs of the MCF's $\underline{g}_i \subset \underline{\underline{G}}$ and $\underline{g}_j \subset \underline{\underline{G}}$ be the two random variables $\underline{x}_i$ and $\underline{x}_j$,

$$\underline{x}_i = \underline{g}_i^T\ \underline{x} \tag{A-11}$$

and

$$\underline{x}_j = \underline{g}_j^T\ \underline{x} \tag{A-12}$$

To prove the lemma, the covariance σ_{ij} between $\underline{x}_i$ and $\underline{x}_j$ will be shown to be 0 for all $i = 1, \ldots, m$ and $j \neq i$:

$$\sigma_{ij} = \overline{\underline{x}_i \underline{x}_j^T}$$

$$= \overline{\left(\underline{g}_i^T \underline{x}\right) \left(\underline{g}_j^T \underline{x}\right)^T}$$

$$= \underline{g}_i^T \overline{\underline{x}\,\underline{x}^T} \underline{g}_j$$

$$= \underline{g}_i^T \underline{\underline{\Omega}}_{xx} \underline{g}_j \qquad \text{(A-13)}$$

Equation A-1 shows that $\underline{g}_i^T \underline{\underline{\Omega}}_{xx} \underline{g}_j = 0$ if $i \neq j$.

Thus, it can be seen that $\sigma_{ij} = 0$. Since this is true for all $i = 1, \ldots, m$ and $j \neq i$, the theorem is proved. The term "linear independence" is used in Lemmas 3, 4, and 5 in a statistical sense.

Q.E.D.

- Lemma 4

The column vectors of the n x m matrix $\underline{\underline{H}}$, when used as prediction MCF's on group $\underline{y}$, give m linearly independent outputs.

This lemma can be proved by following the same procedure as used in Lemma 3 and recalling the $\underline{\underline{H}}^T \underline{\underline{\Omega}}_{yy} \underline{\underline{H}} = |\underline{\underline{\lambda}}|^2$ (Equation A-5).

Q.E.D.

- Lemma 5

An error channel is defined as a random variable obtained from the difference between the outputs of the reference MCF and the prediction MCF. The error channels generated by m such MCF pairs are linearly independent.

Let $\underline{g}_i$, $\underline{g}_j \subset \underline{\underline{G}}$ be two reference MCF's and $\underline{h}_i$, $\underline{h}_j \subset \underline{\underline{H}}$ be two associated prediction MCF's. Then, from Equation 2-1,

$$\begin{Bmatrix} \underline{g}_i \\ -\underline{h}_i \end{Bmatrix}^T \begin{Bmatrix} \underline{x} \\ \underline{y} \end{Bmatrix} = \epsilon_i \tag{A-14}$$

and

$$\begin{Bmatrix} \underline{g}_j \\ -\underline{h}_j \end{Bmatrix} \begin{Bmatrix} \underline{x} \\ \underline{y} \end{Bmatrix} = \epsilon_j \tag{A-15}$$

To prove the lemma, the covariance σ_{ij} between the error channels ϵ_i and ϵ_j will be shown to be 0 for all $i = 1, \ldots, m$ and $j \neq i$:

$$\sigma_{ij} = \overline{\epsilon_i \epsilon_j^T}$$

$$= \begin{Bmatrix} \underline{g}_i \\ -\underline{h}_i \end{Bmatrix}^T \begin{bmatrix} \underline{\underline{\Omega}}_{xx} & \underline{\underline{\Omega}}_{xy} \\ \underline{\underline{\Omega}}_{yx} & \underline{\underline{\Omega}}_{yy} \end{bmatrix} \begin{Bmatrix} \underline{g}_j \\ -\underline{h}_j \end{Bmatrix} \tag{A-16}$$

or

$$\sigma_{ij} = \left(\underline{g}_i^T \underline{\underline{\Omega}}_{xx} - \underline{h}_i^T \underline{\underline{\Omega}}_{yx}\right) \underline{g}_j$$

$$- \left(\underline{g}_i^T \underline{\underline{\Omega}}_{xy} - \underline{h}_i^T \underline{\underline{\Omega}}_{yy}\right) \underline{h}_j \tag{A-17}$$

From Equations A-1 and A-5,

$$\underline{g}_i^T \underline{\underline{\Omega}}_{xx} \underline{g}_j = \underline{h}_i^T \underline{\underline{\Omega}}_{yy} \underline{h}_j = 0$$

Therefore, Equation A-17 simplifies to

$$\sigma_{ij} = -\left(\underline{h}_i^T \underline{\underline{\Omega}}_{yx} \underline{g}_j + \underline{g}_i^T \underline{\underline{\Omega}}_{xy} \underline{h}_j\right) \qquad \text{(A-18)}$$

Equation A-6 and its transpose conjugate show that

$$\underline{g}_i^T \underline{\underline{\Omega}}_{xy} \underline{h}_j = \underline{h}_i^T \underline{\underline{\Omega}}_{yx} \underline{g}_j = 0$$

Substituting this result in Equation A-18 yields $\sigma_{ij} = 0$. This is true for all $i = 1, \ldots, m$ and $j \neq i$.

Q. E. D.

APPENDIX B

FORMULAS FOR CROSSPOWER SPECTRA USED IN MODELS

APPENDIX B

FORMULAS FOR CROSSPOWER SPECTRA USED IN MODELS

- Crosspower spectrum due to directional energy:

$$\Phi_{12}(f) = e^{-j 2 \pi f \cdot \frac{|x|}{|V|} \cos \theta}$$

- Crosspower spectrum due to isotropic energy propagating with fixed speed (cylinder model):

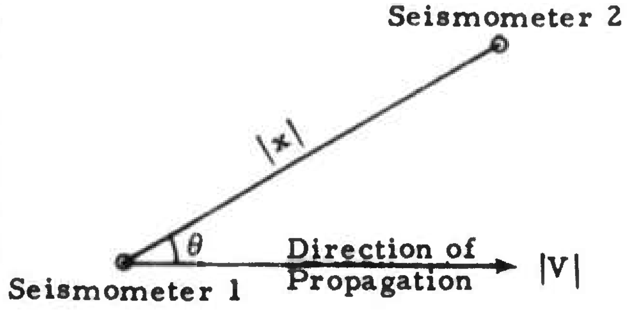
$$\Phi_{12}(f) = J_0\left(2 \pi f \frac{|x|}{|V|}\right)$$

where J_0 represents the zero-order Bessel function.

- Crosspower spectrum due to isotropic energy propagating with speeds above V_e:

$$\Phi_{12}(f) = \frac{V_e}{\pi f \, |x|} \quad J_1\left(\frac{2 \pi f \, |x|}{V_e}\right)$$

where J_1 represents the first-order Bessel function.



$|x|$ = Separation between seismometers

$|V|$ = Propagation speed

θ = Angle between line joining seismometers and propagation direction

f = frequency

Figure B-1

NOTE

The crosspower spectrum in the first case depends on the seismometer separation $|x|$ as well as on the seismometer orientation θ relative to the direction of propagation. In the latter two cases, however, the crosspower spectra are independent of the orientation θ and depend only on the separation $|x|$.

APPENDIX C
SUMMARY OF NOTATION

APPENDIX C
SUMMARY OF NOTATION

T	Conjugate transpose
$\subset$	Contained in
$\mid\ \mid$	Absolute value
$\underline{x}$	Complex multivariate random column vector of m elements $\{x_i \mid i = 1, \ldots, m\}$ which are complex Fourier transforms of the outputs of m seismometers in group $\underline{x}$ at fixed frequency f
$\underline{y}$	Complex multivariate random column vector of n elements $\{y_j \mid j = 1, \ldots, n\}$ corresponding to group $\underline{y}$
$\underline{g}$	Complex column vector of m elements $\{g_i \mid i = 1, \ldots, m\}$ which is the reference MCF associated with group $\underline{x}$
$\underline{h}$	Complex column vector of n elements $\{h_j \mid j = 1, \ldots, n\}$ which is the prediction MCF associated with group $\underline{y}$
ϵ	Error channel, which is the difference between the outputs of MCF $\underline{g}$ and that of MCF $\underline{h}$
$\overline{\lvert\epsilon\rvert^2} = \overline{(\epsilon\epsilon^T)}$	Mean-square-error
$\underline{\underline{\Omega}}_{xx}$	m x m autopower matrix for group $\underline{x}$
$\underline{\underline{\Omega}}_{yy}$	n x n autopower matrix for group $\underline{y}$
$\underline{\underline{\Omega}}_{xy}$	m x n crosspower matrix for groups $\underline{x}$ and $\underline{y}$
$\underline{\underline{\Omega}}_{yx}$	$\underline{\underline{\Omega}}_{xy}^{\ T}$

$\underline{\underline{I}}'$	n x n identity matrix
$\underline{\underline{I}}$	m x m identity matrix
$\lvert\underline{\underline{\lambda}}\rvert^2$	m x m diagonal matrix of eigenvalues $\lvert\lambda\rvert^2$
$\underline{\underline{G}}$	m x m matrix formed by group-coherence MCF's $\{\underline{g}_i \mid i = 1, \ldots, m\}$
$\underline{\underline{H}}$	n x m matrix formed by group-coherence MCF's $\{\underline{h}_j \mid j = 1, \ldots, m\}$
$\rho(\underline{\underline{H}})$	rank of matrix $\underline{\underline{H}}$
min(a, b)	smaller of the two quantities a and b

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13. ABSTRACT

This report investigates the effectiveness of the minimum-power array processing technique in determining seismometer inequalizations. The technique involves partitioning the seismometer array into two groups and designing MCF's for each group so that the mean-square-error between the two MCF outputs is a minimum under the constraint that the output power of one of the MCF's is unity. The two MCF sets are known as the group-coherence filters; the difference between these sets is known as the minimum-power array processor.

Estimates of the noise wavenumber spectrum from the wavenumber responses of the group-coherence filters are distorted due to seismometer inequalization; however, a more reasonable estimate of the noise wavenumber spectrum from the wavenumber response of the minimum-power array processor should be possible because the minimas in the processor's wavenumber response correspond to the wavenumber regions where the wavenumber responses of the two group-coherence MCF's are very similar (e.g., at the peaks of the noise wavenumber power spectrum).

Seismometer inequalization was to be determined from the adjustment in weight and phase required for each filter so that the wavenumber responses of the group-coherence MCF's would agree with a reasonable noise wavenumber spectrum. However, results from the TFO long-noise sample and two synthetic models show that the technique, although excellent for generating maximum coherent channels, lacks the wavenumber resolution desired for studying seismometer inequalization. This latter conclusion is at least true for small arrays such as TFO.

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KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Advanced Array Research						
Minimum-Power Array Processing						
TFO Long-Noise Sample						
Seismometer Inequalization						
Group-Coherence Filters						